

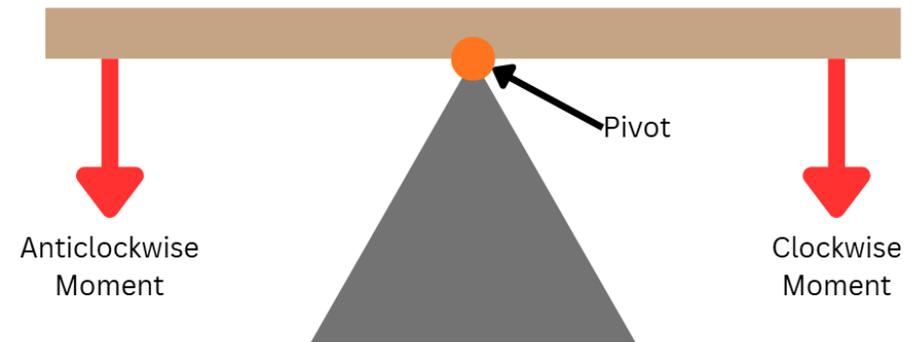
Forces and Motion #3



**UNIVERSITY
CENTRE**

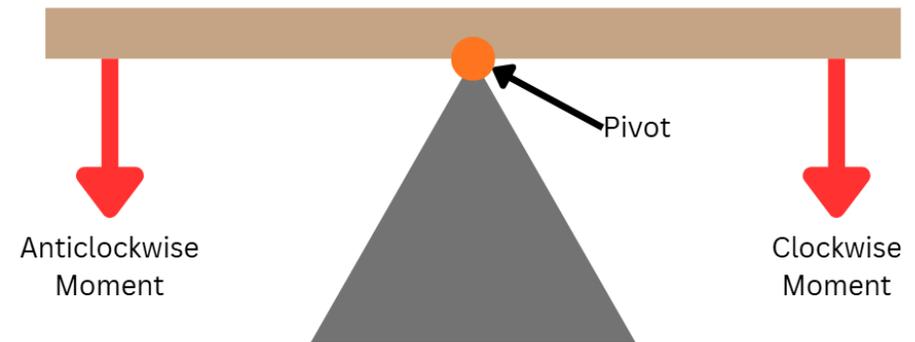
Moments and Torque

- Both moments and torque **mathematically** mean the same thing (a force acting at a distance from pivot)
- Semantically they are very different:
 - **moment** is often used to describe a **bending force**
 - **torque** is used as a **twisting force**
- Both however come from a force which happens a set distance from a pivot and produce a turning action (bending/twisting)



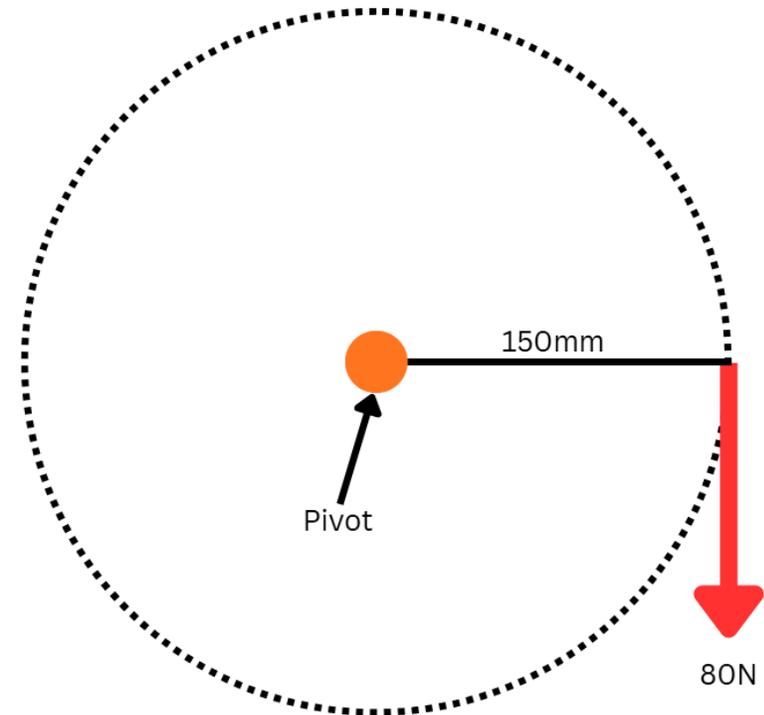
Moments and Torque

- The moment or torque of a force is given by:
- $M = F * d$ or $\tau = F * d$
- Where:
 - M = Moment (Nm)
 - τ = Torque (Nm)
 - F = Force (N)
 - d = perpendicular distance from pivot (m)



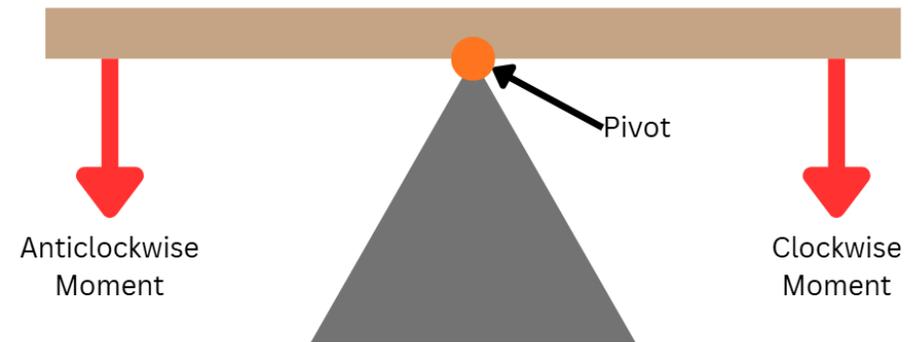
Calculating a moment/torque

- If we have a handle on a crank which has a 150mm radius, and we put 80N of force (clockwise) through that crank what will be the torque generated?
- First, we convert to metres:
 - 150mm = 0.15m
- Then we substitute in our values:
 - $\tau = F * d = 80 * 0.15 = 12Nm$
- We also need to apply direction so its 12Nm clockwise



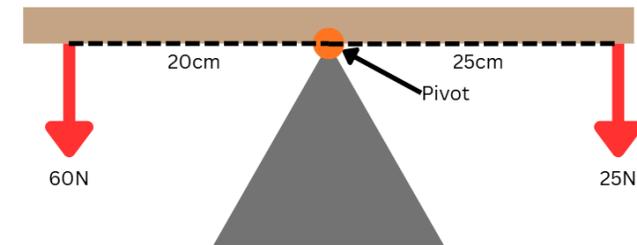
Resultant moment/torque

- When we have multiple forces acting on the same object, we may have to calculate a resultant moment or torque
- To do this we first work out the moment/torque of both, then we add them together, treating the counterclockwise as positive and the clockwise as negative
- $\sum M = \sum M_{anti-clockwise} - \sum M_{clockwise}$
- $\sum \tau = \sum \tau_{anti-clockwise} - \sum \tau_{clockwise}$

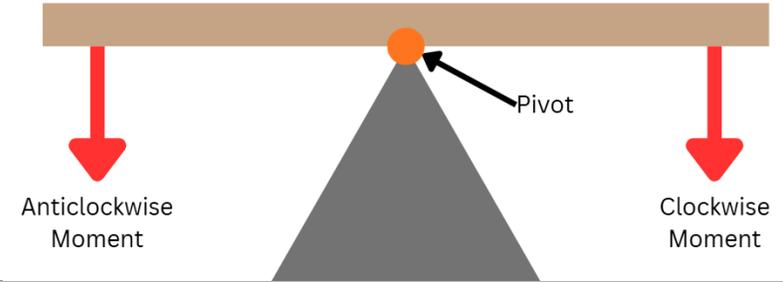


Calculating a resultant moment/torque

- We have a beam with two forces acting on it on either side, one is 20cm away from pivot with a force of 60N and the other is 25cm away from pivot with a force of 25N
- First, we work out the moment of both forces:
 - Anticlockwise = $0.2 * 60 = 12$
 - Clockwise = $0.25 * 25 = 6.25$
- Then we work out the resultant using the equation
 - $\sum M = \sum M_{anti-clockwise} - \sum M_{clockwise}$
 - $\sum M = 12 - 6.25 = 5.75$



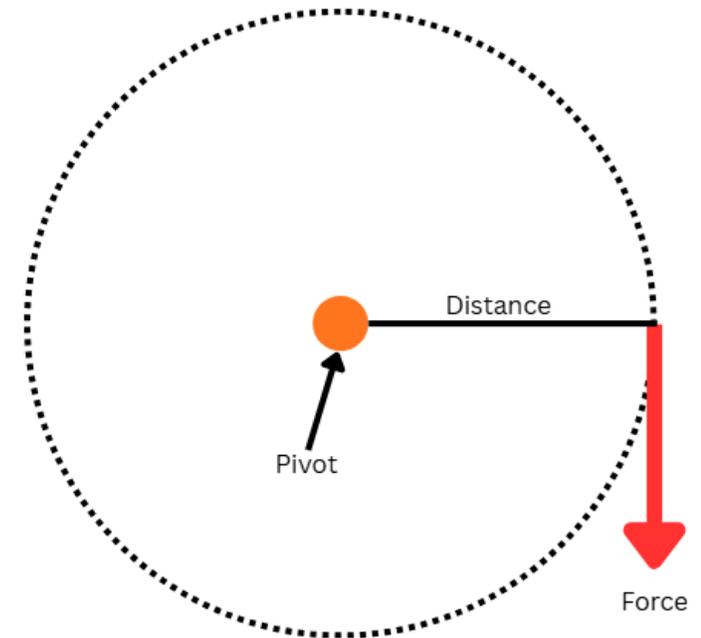
Your turn



1. If we have a handle on a crank which has a **250mm radius**, and we put **75N** of force (anticlockwise) through that crank what will be the torque generated?
2. An engineer is using a spanner to **loosen a nut**. He knows he needs to put in **more than 20Nm** to turn the bolt. If he holds the spanner by the grip (**30cm from pivot**)
 - A. How much force will he need to put in?
 - B. If he can't generate enough force to remove the nut, what would make it easier?
3. A horizontal lifting beam is supported by a pivot at one end. A **150N** motor is mounted **0.8m** from the pivot. On the opposite side of the pivot a technician applies a **200N** upward force using a handle **0.6m** long. Determine the **resultant moment** about the pivot and state the **direction of rotation**.
4. A small crane pivots at point O. A **40N** load is held **0.30 m** from the pivot. Engineers want to balance the arm using a counterweight of **25N** placed on the opposite side of the pivot.
 - A. What distance from the pivot should the counterweight be to achieve equilibrium
 - B. Explain why it might be important to balance the rotation

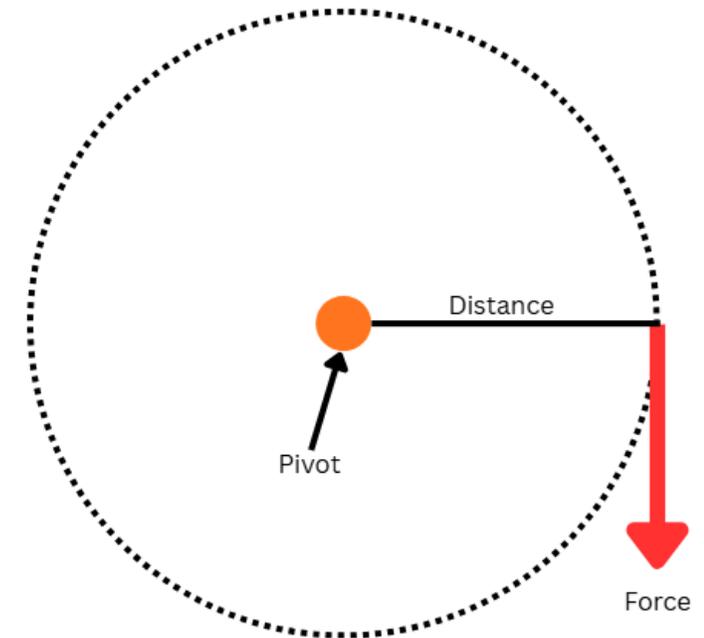
Rotational power

- Power is used to describe how quickly **work is done** or **energy is transferred**
- For rotational systems (like motors or shafts) we use:
- $P = T\omega$
- Where:
 - P = Power (Watts)
 - T = Torque (Nm)
 - ω = Angular velocity (rad/s)



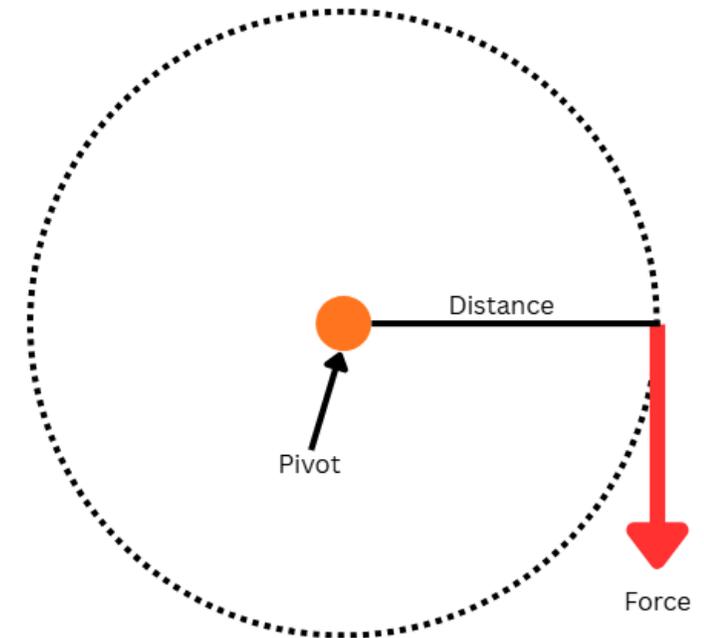
Angular Velocity

- We can calculate angular velocity using regular rpm:
- $\omega = \frac{2\pi N}{60}$
- Where:
 - ω = Angular Velocity (rad/s)
 - N = rotational speed (rpm)
- **Most machines give rpm so its important we convert**



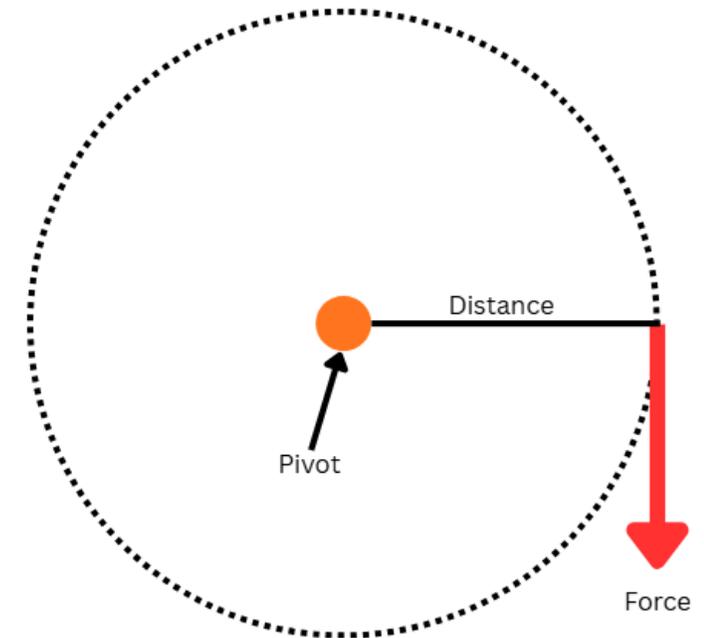
Calculating rotational power

- A motor produces 60 Nm of torque and rotates at 1200 rpm.
- First, we convert to angular velocity:
 - $\omega = \frac{2\pi * 1200}{60} = 125.66 \text{ rad/s}$
- Then we work out the rotational power:
 - $P = 60 * 125.66 = 7539 \text{ W}$
- We can also then convert to kW to make it neater:
 - $7539 \text{ W} = 7.54 \text{ kW}$



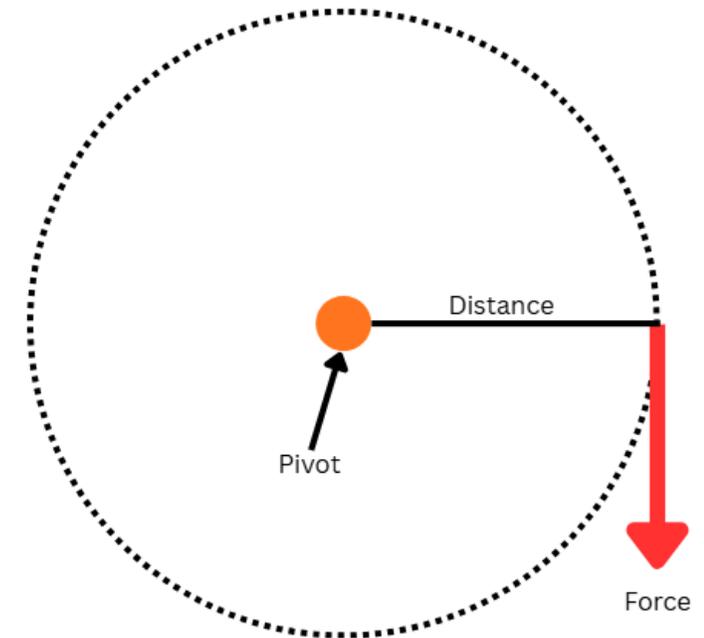
Angular Work Done

- Work can be done when a force causes an object to rotate
- For rotational systems we use:
- $W = T\theta$
- Where:
 - W = Work Done (J)
 - T = Torque (Nm)
 - θ = angular displacement (radians)



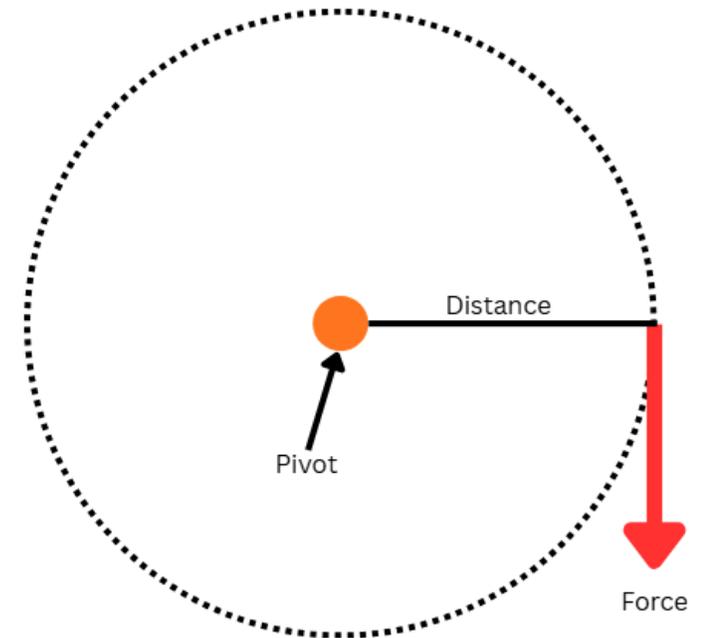
Angular Displacement

- Often displacement is given in revolutions completed, we can convert this to angular displacement to calculate work done
- 1 revolution = 2π radians



Angular Work Done

- Work can be done can also be calculated using power and time
- For rotational systems we use:
- $W = Pt$
- Where:
 - W = Work Done (J)
 - P = Power (W)
 - t = time (seconds)

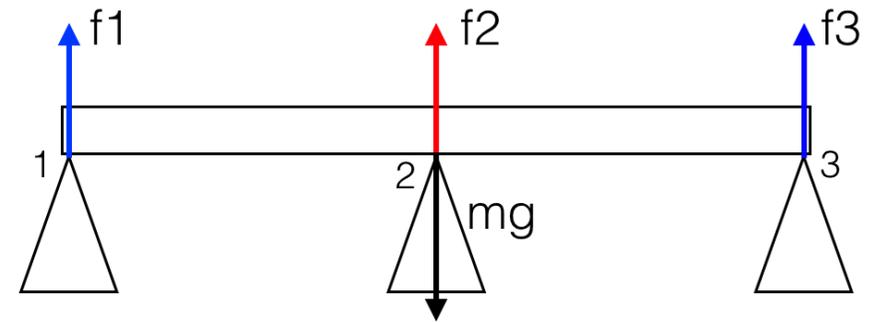


Your turn

1. An electric motor applies a torque of 35Nm to a shaft. The shaft rotates through 8 radians. Calculate the work done by the motor.
2. A machine shaft experiences a torque of 50Nm. The shaft rotates 6 complete revolutions while moving a component. Calculate the work done by the shaft. (Remember: 1 revolution = 2π radians)
3. A motor produces 45Nm of torque while rotating at 900rpm. Calculate the power output of the motor. (You must convert rpm to rad/s first.)
4. A motor applies a torque of 60Nm to a rotating shaft. The shaft rotates at 1000rpm.
 - A. Calculate the power output of the motor.
 - B. Determine the work done by the motor in 12 seconds.

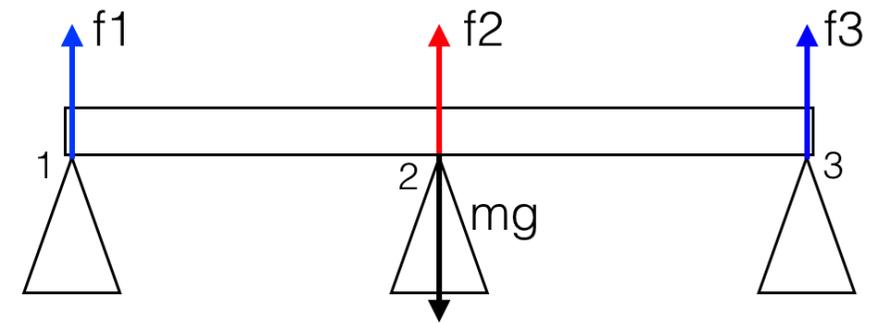
Static Equilibrium Beams

- A structure is in **static equilibrium** when it is **not moving and not rotating**.
- For a beam this means:
 - **The beam is not accelerating vertically**
 - **The beam is not rotating**
- This means 2 conditions must be true:
 - **All upward forces must equal all downward forces.** This ensures the beam does not move up or down. ($\sum F_y = 0$)
 - **The turning effects around any point must balance.** This prevents the beam from rotating. ($\sum M = 0$)



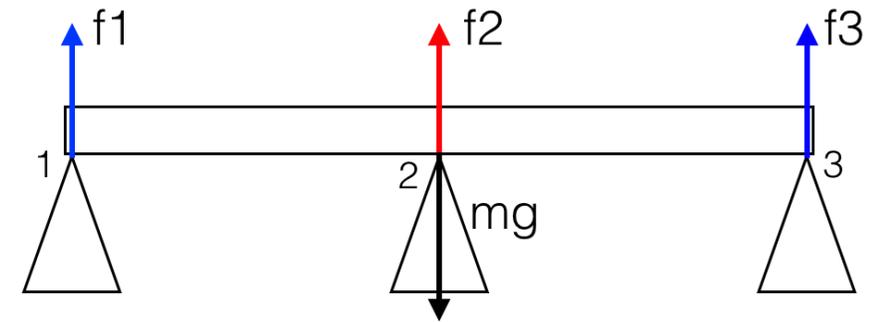
Static Equilibrium Beams

- Most engineering beam problems involve simply supported beams.
- These have two supports:
 - Pin support → provides vertical reaction
 - Roller support → provides vertical reaction
- We label these:
 - R_1 and R_2
 - These are the reaction forces holding the beam up.



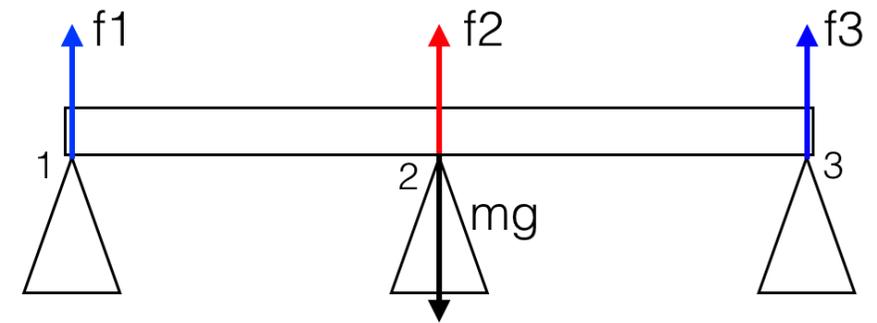
Types of load in a Equilibrium Beam

- Point Load
 - A single concentrated force.
 - Example: a machine placed on a beam.
- A load spread evenly across a distance.
 - Example: weight of flooring or stored materials.
 - To analyse beams, a UDL is replaced with one equivalent force.
- $F = w * L$
- Where:
 - w = load per metre (N/m or kN/m)
 - L = length of load (m)
- The force acts in the centre of the load



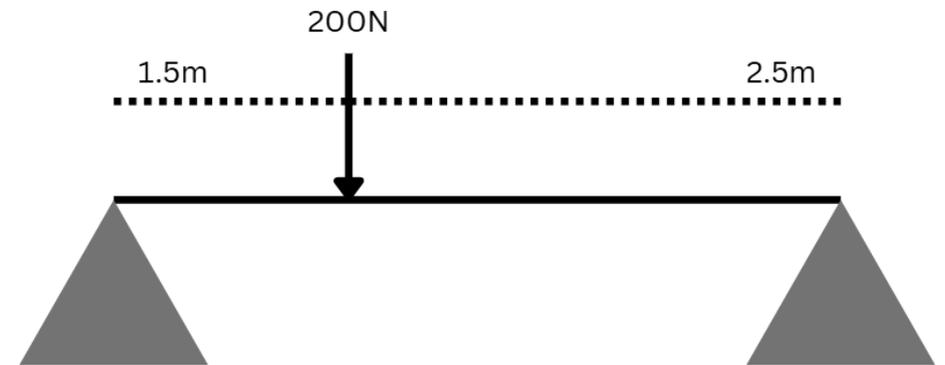
Method for solving Equilibrium Beams

1. Draw the Beam Diagram,
 - Show: Supports, Reaction forces, Loads, Distances
2. Convert distributed load into a single force.
 - Place it at the midpoint of the loaded section
3. Take moments about one support
 - This removes one unknown reaction from the equation
 - For instance $\sum M_a = 0$
 - Solve for the opposite reaction
4. Use vertical force equation
 - $\sum F_y = 0$
 - This allows you to find the second reaction force.



Solving an Equilibrium Beam

- A beam is 4m long and simply supported. A 200N load acts 1.5m from the left support.
- Step 1 is to draw out the system
- Step 2 is to take the moments about the left support
 - $200 * 1.5 = 300Nm$
 - Therefore, reaction at right support must be:
 - $R_2 * 4 = 300 \rightarrow R_2 = \frac{300}{4} = 75N$
- Step 3 is to solve vertical forces:
 - $R_1 + R_2 = 200$
 - $R_1 + 75 = 200$
 - $R_1 = 125N$



Your turn

- A 2.0m horizontal beam is supported by two supports:
 - Support A at the left end
 - Support B at the right end
- Two pieces of equipment are mounted on the beam:
 - A 120 N motor located 0.5 m from Support A
 - A 200 N gearbox located 1.6 m from Support A
- The beam itself weighs **100 N**, acting at its **centre (1.0 m from Support A)**.
 - a) Draw a **free body diagram** of the beam showing all forces and support reactions.
 - b) Calculate the **reaction force at Support B** by taking moments about **Support A**.
 - c) Using vertical force equilibrium, determine the **reaction force at Support A**.